Math 2150 - Homework # 10

Reduction of Order

1. Given below are a second-order linear ODE, a solution y_1 to the ODE, and an interval I that the solution exists on. (i) Use reduction of order to find a second linearly independent solution y_2 to the ODE on I. (ii) Then state the general solution to the ODE on I.

   (a) $x^2y'' - 7xy' + 16y = 0, \quad y_1 = x^4, \quad I = (0, \infty)$

   (b) $x^2y'' + 2xy' - 6y = 0, \quad y_1 = x^2, \quad I = (0, \infty)$

   (c) $xy'' + y' = 0, \quad y_1 = \ln(x), \quad I = (0, \infty)$

   (d) $4x^2y'' + y = 0, \quad y_1 = x^{1/2}\ln(x), \quad I = (0, \infty)$

   (e) $x^2y'' - 20y = 0, \quad y_1 = x^{-4}, \quad I = (0, \infty)$

   (f) $xy'' - (x+1)y' + y = 0, \quad y_1 = e^x, \quad I = (0, \infty)$

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2. Suppose that you know that $y_1 = x^2$ and $y_2 = x^3$ are linearly independent solutions to $x^2y'' - 4xy' + 6y = 0$ on $I = (0, \infty)$.

   (a) Use variation of parameters a particular solution to
   $x^2y'' - 4xy' + 6y = \frac{1}{x}$ on I.

   (b) Give the general solution to $x^2y'' - 4xy' + 6y = \frac{1}{x}$ on I.

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3. Suppose that you know that $y_1 = x$ and $y_2 = x\ln(x)$ are linearly independent solutions to $x^2y'' - xy' + y = 0$ on $I = (0, \infty)$.

   (a) Use variation of parameters a particular solution to
   $x^2y'' - xy' + y = 4x\ln(x)$ on I.

   (b) Give the general solution to $x^2y'' - xy' + y = 4x\ln(x)$ on I.
